∞ IMPULS 国際算数数学授業研究プロジェクト
International Math-teacher Professionalization Using Lesson Study

Grade 5 Mathematics Lesson Plan

Name of the Unit: Let's investigate the way quantities change
Date: June 19, 2019
Teacher: UNO, Naohito

1   Research Theme
Nurturing students who can make connections among their ideas through problem solving.

2   Goal of the unit
Through examinations of two co-varying quantities using tools such as tables, students will understand proportional relationships.

3   Assessment standards
(1) Interest, Eagerness, and Attitude
Students will become interested in the relationships of two co-varying quantities and examine them on their own accord using tools such as tables.

(2) Mathematical Way of Thinking
Students can examine two co-varying quantities using proportional relationship as a viewpoint.

(3) Mathematical Skills
Students can identify proportional relationships when they were given in tables or other formats.

(4) Knowledge and understanding
Students will understand that when one quantity becomes 2, 3, … times as much, the other quantity also becomes 2, 3, … times as much, the two co-varying quantities are in a proportional relationship.

4   Capacities students are beginning to built in this unit
When this unit, "Let's investigate the way quantities change," was taught, students tried to grasp the relationship between two co-varying quantities through activities to summarize the relationship between the quantities in a table and expressing the relationship in equations, using symbols such as ☐ and ○ if necessary. Students examined what pattern they could identify in the change of one quantity as the other quantity changed by manipulating diagrams and representing the changes in a table. Then, students expressed the pattern they identified in equations using symbols such as ☐ and ○. However, there were a number of students who could identify the pattern but had trouble expressing the pattern in an equation. The contents discussed in this unit is a part of a series of topics discussed in Grades 4 through 6, and these topics are developed through a spiral approach. The focus in this unit is to help students build the capacity to represent the relationship between two quantities using an equation by noticing the idea of the change in one quantity

that corresponds to the change of 1 unit in the other - that is, the proportional constant – through activities to record the changing quantities in a table.

As far as students' capacity to communicate and connect their ideas, it was noticed that not a small number of students had difficulty figuring out how to explain their ideas even if they had worthwhile ideas during a lesson. As a result, they simply listened to each other's ideas. We believe students' learning can be facilitated further if we can help them build the capacities to explain their ideas by making use of mathematical terms that describe changes in quantities appropriately and to organize their explanations based on the equation they have written. Moreover, by representing the relationship in equations with symbols symbols such as ☐ and ◯, we want students to realize the usefulness of the equation because by substituting a number in one symbol, we can determine the number for the other symbols as well. This will in turn build the capacity to make use of equations in various situations. In order to achieve this goal, we will design lessons where students realize the values of communicating each other's ideas because mathematical ideas are built up through whole class discussion.

5   Capacities we want to build in this unit (from the perspective of students connecting their ideas)

> Ideal students the mathematics group aims for --- students who can share their reasoning and ideas and who can also feel others' reasoning and ideas

(1) Students will be able to share their ideas based on their observations of the changes in the quantities. By listening to others' ideas and comparing them to their own, students will be able to express themselves more effectively using their own words and use mathematical terms appropriately.

(2) Instructional strategies
- When students are examining a particular proportional relationship, we will ask students to figure out missing values in the table. They will be asked to explain why they chose those values and explain how those values can be viewed as changed from other given values. In this way, students can build their capacity to justify their ideas. [Students will make use of diagrams and equations]
- As we define proportional relationships, we will make sure that students will understand that when one quantity becomes "◯ times as much," the other corresponding quantity also becomes "◯ times as much." Students will be encouraged to use phrases like "proportional relationship" and "◯ times as much" as they explain their ideas. They will also represent the relationship of the quantities in an equation using the symbols such as ☐ and ◯. In this way, they can build their capacity to communicate the rationale for their ideas. [Students will explain their ideas using mathematical terms appropriately]
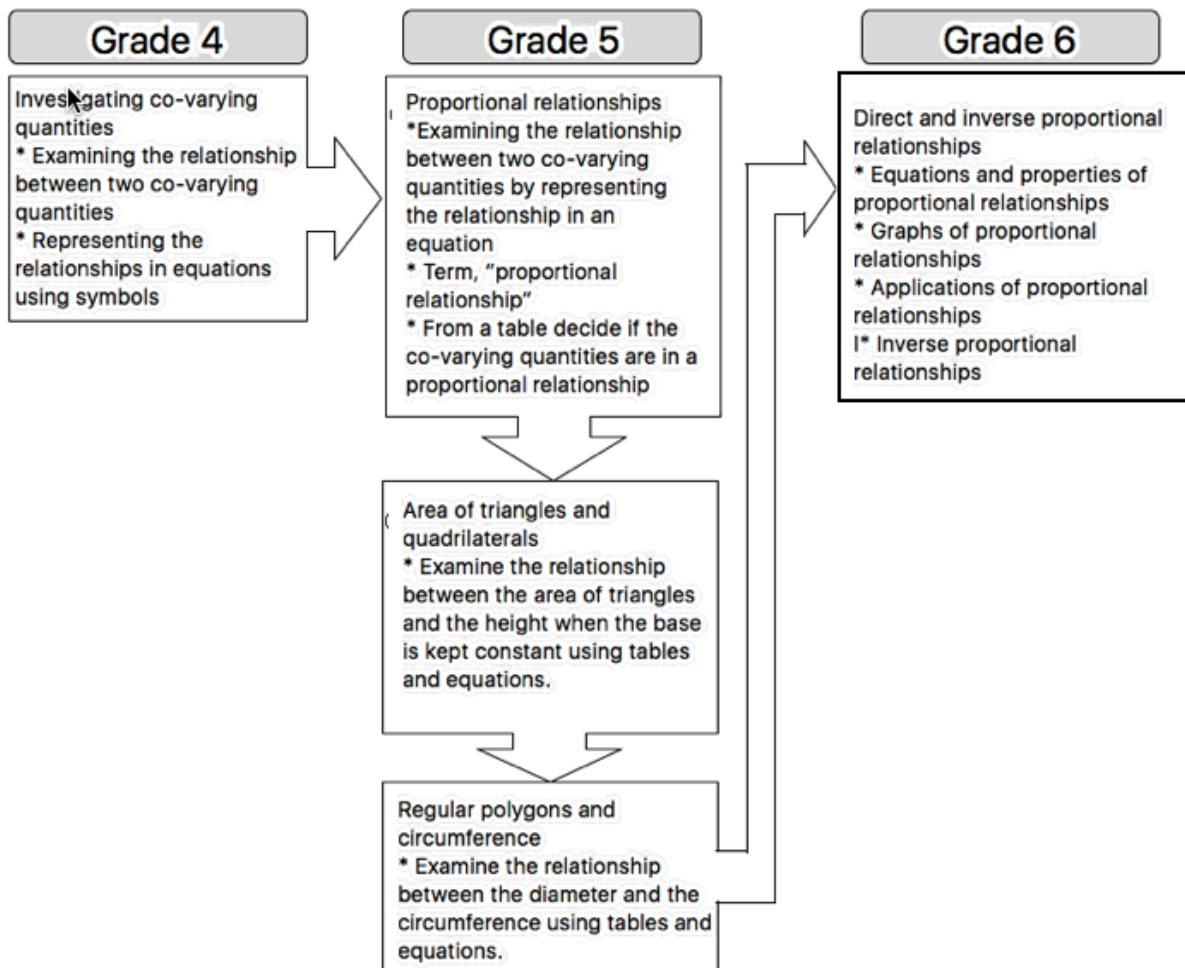
6 About the unit

(1) Unit plan (Total of 3 lessons; today's lesson is lesson 3)

	Goals	Main learning activity	◇ Instructional strategy ☆Evaluation
1	○ Students will understand the relationship between the number of busses and the maximum number of passengers (total). They will understand the term "proportional relationship."	• Students will investigate how the maximum number of passengers (total) will change as the number of busses is made 2, 3, … times as many. • Based on the relationship between the two quantities, students will think about how one of the quantity changes while the other quantity becomes 2, 3, … times as much. • Students will learn the term, "proportional relationship."	◇ Given 2 tables, think about which one shows the relationship when the values of ○ becomes 2, 3, … times as much, the values of △ also becomes 2, 3, … times as much, while writing the times as much relationship in the tables. ☆ Students will become interested in the relationships of two co-varying quantities and examine them on their own accord using tools such as tables. [(1) through observation and students' notebooks] ☆ Students will understand that when one quantity becomes 2, 3, … times as much, the other quantity also becomes 2, 3, … times as much, the two co-varying quantities are in a proportional relationship. [(4) through observation and students' notebooks.
2	○ Through activities to examine the relationship between two co-varying quantities, students can identify and verify a proportional relationship.	• Investigate how the area of room will change when one of its dimensions becomes 2, 3, … times as long. They will summarize that the area of a rectangle is in a proportional relationship with the length of one of its dimensions. • Using the comments and diagrams on pages 141 – 142 in our textbook, students will verify that a proportional relationship may be represented on a number line. • Students will represent a proportional relationship between two quantities in an equation using the symbols □ and ○.	◇ Have students examine the relationship between two co-varying quantities by representing their values in a table. ☆ Students can identify proportional relationships when they were given in tables. [(3) through observation and students' notebooks]

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3	Today's Lesson		
	o Through the activity to examine the relationship between two co-varying quantities in a concrete situation, students can solve problems based on the fact that the two quantities are in a proportional relationship.	• For the two quantities that are in a proportional relationships, students will determine the constant of proportionality and represent the relationship using an equation. • Students will tackle an application problem which requires them to judge if the given co-varying quantities are in a proportional relationship.	◇ Have students think about ways to figure out the missing values in a table based on the proportional relationship. ☆ Students can examine two co-varying quantities using proportional relationship as a viewpoint. [(2) through observation and students' notebooks]

(2) Scope and sequence

Grade 4

Investigating co-varying quantities
* Examining the relationship between two co-varying quantities
* Representing the relationships in equations using symbols

Grade 5

Proportional relationships
*Examining the relationship between two co-varying quantities by representing the relationship in an equation
* Term, "proportional relationship"
* From a table decide if the co-varying quantities are in a proportional relationship

Area of triangles and quadrilaterals
* Examine the relationship between the area of triangles and the height when the base is kept constant using tables and equations.

Regular polygons and circumference
* Examine the relationship between the diameter and the circumference using tables and equations.

Grade 6

Direct and inverse proportional relationships
* Equations and properties of proportional relationships
* Graphs of proportional relationships
* Applications of proportional relationships
I* Inverse proportional relationships

7 Today's lesson (Lesson 3 of 3) --- Grade 5 Classroom 1

Lesson observation points
- Was the emphasis on drawing a table that focuses on the changes of values effective in helping students deciding whether or not the given relationship is a proportional relationship and communicating their ideas to each other?
- Was it effective to review terms and prior learning that are used in this unit for students to communicate their ideas mathematically?

(1) Goal of the lesson
- Students can examine two co-varying quantities using proportional relationship as a viewpoint.

(2) Flow of the lesson

Main learning activity	Anticipated students' reasoning・decision making・expressions	◇ Instructional strategy ☆Evaluation standards and methods
o Grasp the problem situations and organize the quantities in a table to examine if the two quantities are in a proportional relationship. o Students will grasp that the main idea for today's lesson is to examine the relationship between duration and the depth of water by focusing on their dependency relationship.	[Problem situation] We are going to fill the pool with water. For every □ minutes, ○ cm of water is being poured. T: Let's make a table that shows the depth of water at various duration. T: Is the depth of water in a proportional relationship with the duration? C: When the duration becomes 2 times (10 minutes) and 4 times (20 minutes) from 5 minutes, the depth also became 2 times and 4 times. So, I think they are in a proportional relationship. <table><tr><td>T</td><td>0</td><td>3</td><td>5</td><td>9</td><td>10</td><td>12</td><td>15</td><td>20</td><td>36</td></tr><tr><td>D</td><td>0</td><td>9</td><td>15</td><td>27</td><td>30</td><td>36</td><td>45</td><td>60</td><td></td></tr></table> [Goal] Let's investigate if the depth of water and the time are in a proportional relationship and represent their relationship in an equation.	◇ By giving students the information about duration and depth in a ramdom manner, help students realize the usefulness of organizing them in a table. ◇ Confirm that the quantities are in a proportional relationship.
o Think about ways to fill in the blank cell in the table. (Independent problem solving)	T: What can we do to figure out the depth of water at 36 minutes?	◇ If any student seems to be completely stuck, suggest if the idea of a proportional relationship may be used and/or if it is possible to think by how much the depth increases in 1 minute.

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	C1: In 1 minute from 9 minute to 10 minute, the depth increased by 3 cm. So, I think we can keep adding starting with the amount at 20 minutes. At 20 minutes --- 60 cm 60 + 3 + 3 + 3 + …. + 3 = 108	◇ If any student thinks like C1, ask if there is a more efficient way to figure out.
	C2: 36 minutes will be 3 minutes, 3 minutes, 10minutes, and 20 minutes. So, add the depths for those times. 9 + 9 + 30 + 60 = 108	◇ Have students think about the commonality between C3's and C4's ideas by focusing on the ways the values are changing.
	C3: Because in 1 minute from 9 minutes to 10 minute, the depth increased by 3 cm, we can just multiply the amount of increase in the depth in a minute by 36. 3 x 36 = 108	◇ Make sure students understand that C2's, C3's and C5's ideas idea is based on the proportional relationship.
	C4: Because the duration is 3 times as much, make the depth also 3 times as much. 3 x 36 = 108	
	C5: Because these quantities are in a proportional relationship, and because 36 minutes is 3 times as much as 9 minutes, multiply the depth at 9 minutes by 4 27 x 4 = 108	
o Students will explain each other's idea in a small group. o Share the ideas and discuss.	T: Let's share how you thought about the problem. C: The water depth is increasing 3 cm every minute. C: C3 and C4 both used 3 times as much. C: I think C1 could have used multiplication, too. C: I think the "3" is the amount of increase in water depth. C: I think we can represent the relationship in an equation. C: If we use □ for the duration and ○ for the depth, □ x 3 = ○ shows the relationship. C: We can figure out the duration if we know the depth, too. C: If we use the equation, we can figure out unknown durations, too, not just unknown durations.	◇ Encourage students to use phrasing that describes the way quantities are changing, such as "for every minute, the depth is increasing 3 cm." ◇ Help students realize that the situation is related to multiplication. ◇ Have students discuss their ideas in pairs or in groups so that the whole class discussion will lead to the equation. ☆ Students can examine two co-varying quantities using proportional relationship as a viewpoint.

o Have students tackle application problems. ・Are the quantities in a proportional relationship? ・Represent the relationship using an equation using symbols.	T: Can we say these quantities are in a proportional relationship? If they are, how can you represent their relationship in an equation? ① The number, ○, of 90-yen notebooks and the total cost, □ yen. ② When you buy ○ pieces of 20-yen candies and one 100-yen juice, the relationship between the number of candies, ○, and the total cost, □ yen. ③ When you read ○ pages everyday, the number of pages left, □ pages, in a 300-page book. C: In problem (1), if we make the number of notebooks 2, 3, … times as many, the cost will also become 2, 3, … times as much. So, they are in a proportional relationship. C: In problem (2), when the number of candies, ○, becomes 2, 3, … times as many, the cost, □, does not become 2, 3, … times as much. So, they are not in a proportional relationship. C: In problem (3), when the number of pages read, ○, becomes 2, 3, … times as many, the number of pages left, □, does not become 2, 3, … times as many. So, they are not in a proportional relationship.	◇ Have students use specific numbers to examine if the quantities are in a proportional relationship. ◇ If students can write an equation for problem ①, ask them to figure out equations using □ and ○ to represent problems ② and ③also.
o Have students write their learning reflection.	T: Please write your lesson reflection about what you understood in today's lesson and things you thought about. Also, please write your reflection on what you learned in the unit.	

(3) Student evaluation

	Anticipated student's actions
A Very satisfied	Students can examine two co-varying quantities using proportional relationship as a viewpoint and explain their ideas using values and appropriate terms.
B Acceptable	Students can examine two co-varying quantities using proportional relationship as a viewpoint.

＊ Support strategy for students who do not reach B --- help students realize that they can figure out the depth if they use the amount of change in every minute.